numerical data in onerous circumstances. There is nothing else available that does the job so well.

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1. F. L. Bauer, *Nonlinear sequence transformations, Approximation of functions* (H. L. Garabedian, ed.), Elsevier, Amsterdam, 1965, pp. 134–151.
2. J.-P. Delahaye, *Sequence transformations*, Springer Series in Computational Mathematics, Vol. 11, Springer, Berlin, 1988.
3. K. Knopp, *Theory and application of infinite series*, Hafner, New York, 1949.
4. J. R. Schmidt, *On the numerical solution of linear simultaneous equations by an iterative method*, Philos. Mag. **32** (1941), 369–383.
5. D. Shanks, *Non-linear transformations of divergent and slowly convergent sequences*, J. Math. Phys. (M.I.T.) **34** (1955), 1–42.
6. J. Wimp, *Sequence transformations and their applications*, Mathematics in Science and Engineering, Vol. 154, Academic Press, New York, 1981.
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25[65-06, 65D30, 65D32].—TERJE O. ESPELID & ALAN GENZ (Editors), *Numerical Integration: Recent Developments, Software and Applications*, NATO ASI Series, Series C: Mathematical and Physical Sciences, Vol. 357, Kluwer, Dordrecht, 1992, xii+367 pp., $24\frac{1}{2}$ cm. Price $115.00/Dfl.195.00.

Although it is well over 2,000 years ago that Archimedes (287–212 B.C.) started the subject of numerical integration by finding approximations to π, the interest in the subject appears to be far from waning, and this NATO Advanced Research Workshop on Numerical Integration, held in Bergen, Norway over five days in June 1991, attracted 38 delegates from around the world. In all, 34 papers were presented, twenty five appear in full in this volume together with three extended abstracts and one note. The aim of the workshop was to survey recent progress and show how theoretical results have been used in software development and practical applications. This aim has been well achieved. The papers have been subdivided into four sections: "Numerical Integration Rules", "Numerical Integration Error Analysis", "Numerical Integration Applications", and finally "Numerical Integration Algorithms and Software". A complete list of authors and papers is given at the end of this review.

To the reviewer's delight, he found that this volume is dedicated to James Lyness "on the occasion of his 60th birthday". James has been contributing to the subject of numerical integration since his first paper, with John Blatt and David Mustard, was published nearly 30 years ago in the Computer Journal [3]. This Conference did not find James lacking and he describes, in good anecdotal style, his experience involving quadrature over a triangle or quadrilateral when the integrand has a known singularity at a vertex. Under an affine transformation of the region one can get disastrous results. James describes his experiences with this problem.

It is neither possible nor desirable for me to attempt to review, however briefly, every paper, so I shall make a highly personalized selection of papers for brief comment. Let me start with the papers by those authors who were also writing on numerical integration when James Lyness published his first paper. In 1967, Phil Davis and Phil Rabinowitz published the first edition of "Numerical Integration" [1]. This was followed in 1984 by a much enlarged second edition [2]. Both Phils were at this Conference, although Phil Davis' paper on "Gautschi Summation and the Spiral of Theodorus" does not appear in this volume. One can only speculate on the intricate mathematical threads the master was weaving on this occasion. Phil Rabinowitz, together with Will Smith, gave a careful analysis of interpolatory product integration in the presence of singularities both at end points of the interval of integration and at interior points.

It was in 1973 that Frank Stenger surprised us with his paper [4] in which he demonstrated that one can get exponential convergence of quadrature rules based on the trapezoidal rule even when the integrand has singularities. The analytic ideas given in that paper led to the so-called Sinc methods, which have been developed slowly over the years and deserve to be more widely used. Frank Stenger, together with Brian Keyes, Mike O'Reilly, and Ken Parker considered Sinc methods for indefinite integration and thereby extended their use to the solution $y' = F(x, y)$ over an arc $\frown{ab}$ and $y(a)$ given. Bernard Bialecki, on the other hand, considered Sinc methods for the approximate evaluation of Cauchy principal value integrals. David Hunter described a method similar to the Sinc method for the case when the integrand has a singularity close to the interval of integration. In all these cases the errors behave like $a(N)\exp(-(bN)^{1/2})$, where a may depend (weakly) on N and $b > 0$. It is this exponential decay as $N \to \infty$ which makes these methods so attractive.

The section on error analysis had some excellent papers. Picking out just two of them, Walter Gautschi gave, as usual, a scholarly review of remainder estimates for analytic functions, and this was followed by Helmut Brass' beautifully written paper demonstrating the usefulness of approximation theory in this area.

These days, numerical integration for multi-dimensional integrals is of increasing importance, and the last decade has seen considerable growth in this area. It makes use of parts of mathematics which were not required for one-dimensional integrals. Ronald Cools gives an excellent survey of methods for constructing cubature formulae, making use of both invariant theory and ideal theory. This is followed by a paper by Karin Gatermann making use of linear representation theory of finite groups for constructing cubature formulae. Both Ian Sloan and Harald Niederreiter use number-theoretic methods in their respective papers as a basis for the construction of so-called lattice rules for the numerical integration of smooth periodic functions in s dimensions taken over the s-dimensional unit cube. Each paper contains a review of earlier work together with some recent developments undertaken by the respective authors.

But inevitably it is the software implementation of all these rules and associated error analyses which is of greatest interest to applied mathematicians, scientists, and engineers. The last 70 pages are devoted to this topic and of the papers in this section I might pick out for special mention just two. Ronald Cools again, this time with Ann Haegemans, gave a progress report on CUBPACK which, as its name implies, considers adaptive schemes for multi-dimensional integrals. Some good improvements in efficiency over existing schemes is reported for cases when the integrand is either singular or discontinuous. Finally, one of the editors, Terje Espelid, in a brief note reports on a new one-dimensional general-purpose algorithm DQAINT for adaptive quadrature of a function over a collection of intervals and compares his results with those obtained from other schemes.

Multi-dimensional integration and automatic integration occupied 20% and 10% respectively of [2]. This volume devotes about half of the papers to multidimensional integration and a further quarter to automatic integration. This points out the need for yet a third edition of the two Phils' magnificently organized and presented books. I understand that there are no plans for this at present, but perhaps a younger person might make his or her name in the field by undertaking this task in collaboration with the other two. There is excellent precedence for doing this with mathematical books. Two which come to mind are Watson in "Whittaker and Watson" and Jaeger in "Carslaw and Jaeger".

Browsing through these proceedings nearly 12 months after the event and from the other side of the world, one gets the impression that this has been an excellent Workshop, and the organizers are to be congratulated on their efforts. This volume does provide a good reference to the current state of the art and no self-respecting numerical analyst should be without access to these Proceedings. So if you cannot afford to put this volume on your own shelves, at least get your library to buy a copy.

Contributions: R. Cools, A survey of methods for constructing cubature formulae; K. Gatermann, Linear representations of finite groups and the ideal theoretical construction of G-invariant cubature formulas; H. J. Schmid and H. Berens, On the number of nodes of odd degree cubature formulae for integrals with Jacobi weights on a simplex; K.-J. Förster, On quadrature formulae near Gaussian quadrature; I. Sloan, Numerical integration in high dimensions—the lattice rule approach; H. Niederreiter, Existence theorems for effective lattice rules; B. Bialecki, SINC Quadratures for Cauchy principal value integrals; P. Rabinowitz and W. E. Smith, Interpolatory product integration in the presence of singularities: L_p theory; D. B. Hunter, The numerical evaluation of definite integrals affected by singularities near the interval of integration; N. I. Ioakimidis, Application of computer algebra software to the derivation of numerical integration rules for singular and hypersingular integrals; W. Gautschi, Remainder estimates for analytic functions; H. Brass, Error bounds based on approximation theory; K. Petras, One sided L_1-approximation and bounds for Peano kernels; R. Cariño, I. Robinson, and E. De Doncker, An algebraic study of the Levin transformation in numerical integration; G. Hämmerlin, Developments in solving integral equations numerically; C. Schwab and W. L. Wendland, Numerical integration of singular and hypersingular integrals in boundary element methods; J. N. Lyness, On handling singularities in finite elements; K. Hayami, A robust numerical integration method for 3-D boundary element analysis and its error analysis using complex function theory; J. Berntsen, On the numerical calculation of multidimensional integrals appearing in the theory of underwater acoustics; A. Genz, Statistics applications of subregion adaptive multiple numerical integration; F. Stenger, B. Keyes, M. O'Reilly, and K. Parker, The Sinc indefinite integration and initial value problems; P. Keast, Software for integration over triangles and general simplices; R. Cariño, I. Robinson, and E. De Doncker, An algorithm for automatic integration of certain singular functions over a triangle; R. Cools and A. Haegemans, CUBPACK: Progress report; E. De Doncker and J. Kapenga, Parallel cubature on loosely coupled systems; M. Beckers and A. Haegemans, Transformation of integrands for lattices rules; T. O. Espelid, DQAINT: An algorithm for adaptive quadrature over a collection of finite intervals; C. Schwab, A note on variable knot, variable order composite quadrature for integrands with power singularities; A. Sidi, Computation of oscillatory infinite integrals by extrapolation methods.

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1. P. J. Davis and P. Rabinowitz, *Numerical integration*, Blaisdell, Waltham, MA, 1967.

2. ——, *Numerical integration*, Academic Press, New York, 1984.

3. D. Mustard, J. N. Lyness and J. M. Blatt, *Numerical quadrature in n dimensions*, Comput. J. **6** (1963), 75–87.

4. F. Stenger, *Integration formulae based on the trapezoidal formula*, J. Inst. Math. Appl. **12** (1973), 103–114.

26[49–02, 49J15, 49M37].—K. C. P. MACHIELSEN, *Numerical Solution of Optimal Control Problems with State Constraints by Sequential Quadratic Programming in Function Space*, CWI Tract, Vol. 53, Centre for Mathematics and Computer Science, Amsterdam, 1988, vi+214 pp., 24 cm. Price: Softcover Dfl.59.00.

The aim of this book is to present the application of SQP (the sequential quadratic programming algorithm) to the optimal control of ordinary differential equations with state and control constraints.

The book is structured as follows. Chapter 1 is a brief introduction. Chapter 2 presents a theory of first- and second-order optimality conditions of abstract optimization problems in Banach spaces. The optimality conditions for optimal control problems are presented in Chapter 3, and Chapter 4 presents the principle of SQP for abstract optimization problems and optimal control (without discretization). Chapter 5 discusses the optimality conditions of the quadratic subproblems. The numerical resolution of the quadratic problems is discussed in Chapter 6; Chapter 7 presents some interesting examples in flight mechanics and servo systems. Final remarks are made in Chapter 8. The appendix is devoted to some numerical questions.

Dealing with the subject of this book is a difficult task because one has, on one hand, to present some results of optimization in infinite-dimensional spaces that use the most sophisticated tools of functional analysis, and on the other hand deal with some algorithmic and numerical questions which are rather involved. It is also worth noting that no other book (to the knowledge of the reviewer) covers such a vast domain.

The book seems to have been written very rapidly, and some drawbacks are apparent. For instance, the abstract optimization theory of Chapter 2 is very heavy, probably because of the desire of the author to present several qualification conditions. Also the second-order necessary condition of Theorem 2.14 says that the Hessian of the Lagrangian is nonnegative for a critical direction,